

Assignment 4

1. Suppose $f : [a, b] \rightarrow \mathbb{R}$ is uniformly continuous. Show that f is bounded.
2. Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is uniformly continuous and $\lim_{|x| \rightarrow \infty} f(x) = 0$. Show that f is bounded.
3. Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is uniformly continuous. Show that $|f(x)| < A + B|x|$ for some positive constants A and B .
4. Find a couple of sequences of polynomials each converging to a non-polynomial in sup norm over $[0, 1]$ (avoid the one given in the notes).
5. Consider $\mathcal{C}[a, b]$, and a sequence (p_n) such that $p_n = x^n$. Find for what values of a, b , the sequence (p_n) is a Cauchy sequence.